C.O.E office 27/11/24 (M) KMV - II

Exam Code: 112415
(30)

Paper Code: 5177

Programme: Bachelor of Science (Honours) Mathematics Semester-V

Course Title: Number Theory

Course Code: BOML-5331

Time Allowed: 3 Hours **Max Marks: 80**

Note: Attempt five questions, selecting at least one question from each section. The fifth question may be attempted from any section. Each question carries equal 16 marks.

SECTION-A

1(a)	Determine all solutions in positive integers of Diophantine of equation $18x+5y=48$.	8
(b)	Show that is $3^{287}-3$ divisible by 23.	8
2(a)	Prove that an integer is divisible by 4 if and only if the number formed by its ten units is divisible by 4.	8
(b)	Define a *Reduced Residue System (RRS) modulo m*. Form a RRS modulo 12.	8

SECTION-B

3.	State and prove Chinese Remainder Theorem for a set of simultaneous linear congruences.	16
4(a)	If $\gcd(a,133)=\gcd(b,133)=1$, then using Fermat's theorem prove that $a^{18}\equiv b^{18}(\text{mod}\,133)$	8
(b)	Solve linear congruence $13x\equiv 3(\text{mod}\,47)$.	8

SECTION-C

5.	State and prove Wilson's Theorem.	16
6(a)	Find a positive integer k such that $\phi(2k)=\phi(k)$.	8
(b)	Show that $a^{560}\equiv 1(mod\,560)$ if $\gcd(a,561)=1$, however 561 is not a prime.	8

SECTION-D

7(a)	State Mobius Inversion Formula. Verify this formula for a positive integer 24.	10
(b)	Prove that the sum of divisors function $\sigma(n)$ is a multiplicative function.	6
8(a)	Find the highest power of 3 dividing $\underline{|533}$.	8
(b)	Show that $\prod_{d\mid n} d = n^{\frac{\tau(n)}{2}}$, where $\tau(n)$ denotes the number of positive divisors of n.	8

Exam Code: 112415 Paper Code: 5178
(30)

Programme: Bachelor of Science (Honours) Mathematics Semester-V

Course Title: Discrete Mathematics

Course Code: BOML-5332

Time Allowed: 3 Hours Max Marks: 80

Note: Attempt five questions in all, selecting at least one question from each section. Fifth question can be attempted from any section. Each question carries equal 16 marks.

Section-A

Q.1 State and prove the following laws in a Boolean algebra

(i) Idempotent Law (ii) Boundedness Law (iii) Absorption Law (iv) Involution law (16)

Q.2 (a) Use K rnaugh map to find the minimal sum for

$$f(x,y,z,t) = xy' + xyz + x'y'z' + x'yzt'$$

(b) Consider the Boolean Function $f(x,y,z) = (xy+z)(x'+yz')(x'+z)$. solve it algebraically and hence draw its circuit diagram.

(c) Check whether the expressions are equal or not?

$ab + a'b' = (a'+b)(a+b')$

(d) Simplify the Boolean expression $f(x,y,z) = x'z + yz + yz'$ and write the minterm normal form.

(4,4,4,4)

Section-B

Q.3 (a) Give an example of 3- regular graph . Draw its Complement and prove that it is also regular .

(b) Prove that the minimum no. of edges in a connected graph with n vertices is n-1.

(c) Draw the graph with following adjacency matrix

$$\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 2 & 3 \\ 2 & 3 & 0 & 1 \\ 3 & 2 & 1 & 0 \end{bmatrix}$$

(4,8,4)

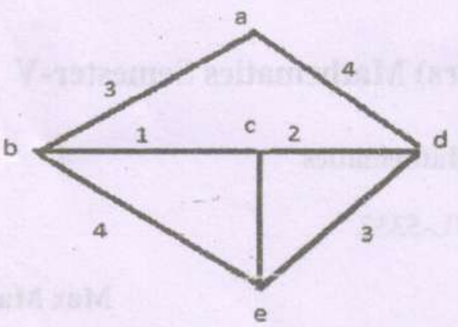
Q.4 (a) Prove that in a connected planar graph G with V vertices , E edges and R regions then V-E+R=2

(b) State and prove the Euler Theorem. (8,8)

Section-C

Q.5 (a) Prove that a planar graph G is five colourable .

(b) Using Kruskal's algorithm , Find Minimal spanning tree for the following graph :



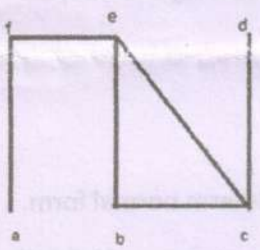
(8,8)

Q.6 (a) Represent the expression as a binary tree and write the prefix and post fix forms of the expression

$((A * B) - (C \uparrow D)) + (E / F)$

(b) Prove that A graph is a tree iff it is minimally connected

(c) Find all Spanning trees of the graph shown in the following figure:



(5,6,5)

Section-D

Q.7 (a) What are the Quantifiers ? Explain the types of quantifiers with the help of examples .

(b) Assume the value of $p \to q$ is false . Determine the value of $(\sim p \vee \sim q) \to q$ (10,6)

Q.8 (a) Using truth table Check whether $p \leftrightarrow \sim q$ logically imply $p \to q$ or not ?

(b) Check the validity of the argument

If I study , then I will pass examination. If I do not go to picnic , then I will study. But I failed examination. Therefore, I went to picnic. (8,8)

C.O.E Office 9/12/24 (M) KMV-II

Exam Code: 112415 **Paper Code: 5179**
(30)

Programme: Bachelor of Science (Honours) Mathematics Semester-V

Course Title: Linear Integral Equations

Course Code: BOML-5333

Time Allowed: 3 Hours **Max Marks: 80**

Note: Attempt five questions in all, selecting at least one question from each section. Fifth question can be attempted from any section. Each question carries 16 marks.

Section-A

1(a) Solve $y(x) = x + \int_0^x (t-x)y(t)\,dt$ with the help of resolvent kernel.

(b) What are the different kinds of Kernel. Explain each with the help of an example.

[8,8]

2(a) Define Volterra Integral Equation of second kind. Convert an initial value problem $y'' - 3\,y' + 2\,y = 4\sin x$ with initial conditions $y(0) = 1, y'(0) = -2$ to a Volterra Integral equation of the second kind.

(b) Solve using the method of successive approximations by taking $\emptyset_0(x) = 1$

$$\emptyset(x) = 1 + x - \int_0^x \emptyset(t)\,dt$$

[8,8]

Section-B

3(a) Determine the resolvent kernel of the Fredholm integral equation with the kernel:

$$K(x,t) = (1+x)(1-t):\ a = -1, b = 1$$

(b) Solve the integral equation: $g(s) = (2s-\pi) + 4\int_0^{\pi/2} \sin^2 s\ g(t)\,dt$

[8,8]

4 (a) Solve: $y(s) = f(s) + \lambda \int_0^1 e^{s-t}\ y(t)dt$

(b) Determine the eigen values and eigen functions of the homogeneous integral equation:

$$\emptyset(x) = \lambda \int_0^1 (2x\xi - 4x^2)\emptyset(\xi)d\xi$$ [8,8]

Section-C

5(a) What is an Integro-differential equation. Solve the Integral equation:

$$\emptyset'(x) = x + \int_0^x \cos t \; \emptyset(x-t)\, dt, \emptyset(0) = 4$$

(b) Solve the integral equation:

$$\emptyset(t) = \int_0^t K(t^2 - x^2)\; \emptyset(x)\, dx, x > 0$$

[8,8]

6(a) Solve the Abel's integral equation: $f(x) = \int_0^x \frac{\emptyset(\xi)}{(x-\xi)^\alpha}\, d\xi \;\; where\; 0 < \alpha < 1$

(b) Define Integral equation of convolution type. Solve:

$$y(x) = x^2 + \int_0^x \sin(x-\xi) y(\xi) d\xi$$

[8,8]

Section-D

7(a) Construct Green's function for the equation $\frac{d^2y}{dx^2} + \mu^2 y = 0$ with the conditions

$$y(0) = y(1) = 0$$

(b) Transform the problem $\frac{d^2y}{dx^2} + y = x, y(0) = 1,\; y'(1) = 0$ to an integral equation.

[8,8]

8(a) Determine the Green's function $G(x,\xi)$ for the differential equation

$$\left[\frac{d}{dx}\left(x\frac{d}{dx}\right) - \frac{n^2}{x}\right] u = 0 \text{ with the conditions } u(0) = 0, u(1) = 0$$

(b) Solve the boundary value problem using Green's function

$$\frac{d^4y}{dx^4} = 1, y(0) = y'(0) = y''(1) = y'''(1) = 0$$

[8,8]

Exam Code: 112415 Paper Code: 5180

(30)

Programme: Bachelor of Science (Honours) Mathematics

Semester-V

Course Title: Riemann Integration

Course Code: BOML-5334

Time Allowed: 3 Hours **Max Marks: 80**

Note: Attempt five questions in all, selecting at least one question from each section. Fifth question can be attempted from any section. Each question carries equal 16 marks.

Section-A

1.(a) State and prove Necessary and Sufficient conditions for Riemann Integrability. (8)

(b) Show that $f(x) = sinx$ is Integrable on $[0,\frac{\pi}{2}]$ and $\int_0^{\frac{\pi}{2}} sinx dx = 1$ (8)

2.(a) Show that $f(x) = \begin{cases} 1, x \in Q \\ 0, x \in R - Q \end{cases}$ is not R-integrable on [0,1] (8)

(b) Let f is bounded function defined on [a,b] and let P be a partition of [a,b]. If P' is the refinement of P, then prove that $L(P,f) \leq L(P',f) \leq U(P',f) \leq U(P,f)$ (8)

Section-B

3.(a) Prove that if a function f defined on [a,b] is bounded and has a finite number of points of discontinuity, then f is Riemann Integrable. (8)

(b) If f_1 and f_2 are two R-integrable function on [a,b] , then prove that $f_1 f_2$ is R-integrable on [a,b]. Is Converse true? (8)

4.(a) State and prove Second Mean value theorem of integral calculus. (8)

4. (b) Show that $\frac{\pi^2}{9} \leq \int_{\pi/6}^{\pi/2} \frac{x}{sinx} dx \leq \frac{\pi^2}{6}$

Section-C

5. State and prove Dirchlet's test for convergence of improper integral and hence show that $\int_e^\infty \frac{\log x \sin x}{x} dx$ is convergent at ∞. (16)

6(a) Examine the convergence and divergence of $\int_0^\infty (\frac{1}{1+x} - e^{-x}) \frac{dx}{x}$. (8)

(b) Show that $\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1} dx$ is convergent for positive values of m and n. (8)

Section-D

7. Define Gamma function with the help of an example and prove that $\beta(m,n) = \frac{\tau(m)\tau(n)}{\tau(m+n)}$ $where\ m > 0\ and\ n > 0$ (16)

8(a) Express $\int_0^2 (8-x^3)^{\frac{-1}{3}} dx$ in terms of Beta function. (8)

(b) Show that $\int_0^{\frac{\pi}{2}} \sqrt{tan\theta} d\theta = \frac{\pi}{\sqrt{2}}$ (8)

Exam Code: 112415 (30)

Paper Code: 5181

Programme: Bachelor of Science (Honours)

Mathematics Semester-V

Course Title: Metric Spaces

Course Code: BOML-5335

Time Allowed: 3 Hours

Max Marks: 80

Attempt five questions in all selecting at least one question from each Section. The fifth question can be attempted from any section. Each question carries equal 16 marks.

Section — A

1. a) Prove that the arbitrary intersection of closed sets is closed. What about their union ? Justify. 8

   b) If (X, d) is a metric space. Let $d_1\ (x, y) = \min\{1, d(x, y)\}, \forall\ x, y \epsilon X$. Then $d,\ d_1$ are equivalent metrics. 8

2. a) A set A is open iff $A = Int.(A)$. 8

   b) A finite union of nowhere dense sets in a metric space is a nowhere set. What about countable union of nowhere dense sets ? Justify with example. 8

Section-B

3. a) State and prove Heine-Boral Theorem. 10

   b) Let $K \subseteq Y \subseteq X$, then K is compact relative to X iff K is compact relative to Y. 6

4. a) The closure of a connected set is connected set in a metric space (X, d). 8

   b) Intervals and only intervals are connected subsets of Real numbers. 8

Section-C

5. a) The set of all sub-sequential limits of a sequence $< x_n >$ in a metric space form a closed subset in X. 8

   b) A Cauchy sequence in a metric space is convergent iff it has convergent subsequence. 8

6. a) Prove that discrete metric space is complete. 6

   b) State and prove Cantor's Intersection Theorem. 10

Section-D

7. a) Let (X, d_1) and (Y, d_2) be two metric spaces and let $f: X \to Y$ be a mapping. Then f is continuous iff $\overline{f^{-1}(F)} \subseteq f^{-1}\,\overline{(F)}, \forall F \subseteq Y$. 8

   (b) Prove that continuous image of a connected set is connected. 8

8. a) Let f be a continuous mapping of a compact metric space X into a metric space Y. Then f is uniformly continuous. 10

   (b) Give an example of a function defined on real line, which is discontinuous at every point. 6