C.O.E office 28/11/24 (EVE) KIMV-II

Exam Code: 225701 **Paper Code: 1217**

Master of Science (Mathematics) Semester I

Course Title: Real Analysis

Course Code: MMSL-1331

Time: 3 Hours **Max. Marks: 70**

Note: Attempt five questions, selecting one question from each section. The fifth question can be attempted from any section. Each question carries 14 (7+7) marks.

Section-A

Q-1 (a) Define the Space R^3,and prove that it is a metric Space.

(b) Prove that the interval [0,1] is uncountable .

Q-2 (a) Prove that closure of a set is always a closed set and it is the smallest closed set containing that set.

(b) State and Prove Weierstrass theorem for R^K

Section-B

Q-3 (a) Define separated sets and prove that a set is disconnected if it is the union of two non empty separated sets

(b) Find out all the components of a discrete metric Space. Also find the component of R. Also prove that every component is closed but need not open.

Q-4 (a) Prove that a metric space is compact iff any sequence in X has a convergent subsequences. Also find the convergent sequence in a discrete metric space.

(b) Define diameter of a set along with nth tail of the sequence. Prove that diameter of closure of E is same as diam of E in metric Space (X,d).

Section-C

Q-5 (a) Define contraction mapping .Give an Example of the same and prove that contraction mapping is Uniformly continuous

(b) State and prove Banach Fixed point theorem

Q-6 (a) Show that a continuous function f of a metric space X into a metric space Y takes convergent sequences into convergent sequences. Also give an example of a function which is continuous at single point of its domain

(b) Give an example of a function which is uniformly continuous. Justify your answer. And prove that every function defined on a discrete metric space is continuous

Section-D

Q-7 (a) State and Prove W.M. test for uniform convergence of series of functions

(b) Test for uniform convergence and term by term integration of the series $\sum_{n=1}^{\infty} \frac{x}{(n+x^2)^2}$. Also prove that $\int_0^1 \left(\sum_{n=1}^{\infty} \frac{x}{(n+x^2)^2}\right) dx = \frac{1}{2}$.

Q-8 (a) Show that the sequence <fŋ> where fŋ(x) = $\frac{nx}{1+n^2x^2}$, is not uniformly convergent on any interval containing '0' by M(n) test

(b) Prove the Weierstrass Approximation Theorem

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Exam Code: 225701 **Paper Code: 1217**

Programme: Master of Science (Mathematics)

Semester - I

Course Title: Complex Analysis

Course Code: MMSL-1332

Time Allowed: 3 Hours **Max. Marks: 70**

Note: Attempt five questions in all, selecting atleast one question from each section. Fifth question may be attempted from any section. Each question carries 14 marks.

Section-A

1(a) Prove that $u = x^2 - y^2, v = \frac{-y}{x^2+y^2}$, then both u and v satisfy Laplace's equation, but $u + iv$ is not an analytic function of z.

(b) Define Harmonic function. Show that a harmonic function satisfies the formal differential equation $\frac{\partial^z u}{\partial z \partial \bar{z}} = 0$.

2(a) Using Cauchy's Integral formula, Calculate $\int_C \frac{e^{az}}{(z-\Pi i)} dz$, where C is the ellipse $|z-2| + |z+2| = 6$.

(b) If $u + v = \frac{2\sin 2x}{e^{2y}+e^{-2y}-2\cos 2x}$ and $f(z) = u + iv$ is analytic function of $z = x + iy$, find $f(z)$ in terms of z.

Section-B

3(a) State and prove Liouville's theorem.

(b) Show the image of the infinite strip $\frac{1}{4} \leq y \leq \frac{1}{2}$ under the transformations

$w \leq \frac{1}{z}$. Also show the regions graphically.

4(a) Find the image of the circle $|z-2| = 2$ under the Mobius transformations
$w = \frac{z}{z+1}$.

(b) Find a bilinear transformation which maps the circle $|w| \le 1$ into a circle $|z-1| < 1$ and maps $w = 0$ *and* $w = 1$ respectively into $z = \frac{1}{2}$, z=0.

Section-C

5 State Taylor's theorem and Laurent's theorem and obtain the Taylor's or Laurent's series which represent the function $f(z) = \frac{1}{(1+z^2)(z+2)}$, when

(i) $|z| < 1$ (ii) $1 < |z| < 2$ (iii) $|z| > 2$

6(a) State and prove Cauchy's Residue Theorem.

(b) Use Rouche's Theorem to show that the equation $z^5 + 15z + 1 = 0$ has one root in the disc $|z| < \frac{3}{2}$ and four roots in the annulus $\frac{3}{2} < |z| \le 2$.

Section-D

7(a) Show that $z = a$ is an isolated essential singularity of the function $\frac{e^{\frac{c}{z}-a}}{e^{\frac{c}{z}}-1}$.

(b) Show that $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta} = \frac{2\pi}{\sqrt{3}}$.

8(a) Show by Contour integration that $\int_0^{\infty} \frac{1}{(1+x^2)} dx = \frac{\pi}{2}$.

(b) Show by Contour integration that $\int_0^{\infty} \frac{\sin mx}{x} dx = \frac{\pi}{2}$, $m > 0$.

Exam Code:225701 **Paper Code: 1219**

Programme: Master of Science (Mathematics)
Semester-I
Course Title: Algebra-I
Course Code: MMSL-1333

Time Allowed: 3 Hours **Maximum Marks:70**

Attempt five questions, selecting one question from each section. The fifth question may be attempted from any Section. Each question carries equal marks. (14)	Marks
Section A	
1. (a) Prove that the set of four 4^{th} root of unity form a finite abelian group of order four under ordinary multiplication as composition.	7
(b)State and prove Lagrange's theorem for finite groups.	7
2. (a) Prove that every subgroup of a cyclic group is cyclic.	5
(b) Prove that Z(G) of a group G is a normal subgroup of G.	5
(c) Determine all the generators of a cyclic group of order 28.	4
Section B	
3. (a) For a finite group G, prove that $O(G) = \sum \frac{O(G)}{O(N(a))}$ where the sum runs over one element a of each conjugate class.	7
(b) State and prove Fundamental theorem of Homomorphism.	7
4.(a) Prove that A_4 has no subgroup of order 6.	7
(b) Prove that A_n is a normal subgroup of S_n.	7
Section C	
5.(a) State and prove Sylow's second theorem.	7
(b) Every finite group has atleast one composition series.	7

6.(a) State and prove Jordan Holder theorem.	7
(b) Show that group of order 40 is not simple.	7
Section D	
7. (a) Define internal direct products.	10
(b) Show that the group Z_8 cannot be written as the direct sum of two non-trivial subgroups.	4
8. (a) Show that the direct product of two cyclic groups G and G' is a cyclic group iff $\left(O(G), O(G')\right) = 1$	7
(b) Write $Z_2 \times Z_2$. Is $Z_2 \times Z_2$ a cyclic group.	7

Exam Code: 225701 **Paper Code: 1220**

Master of Science (Mathematics) Semester I

Course Title: Mechanics - I

Course Code: MMSL-1334

Time: 3 Hours **Max. Marks: 70**

Note: Attempt five questions, selecting one question from each section. The fifth question can be attempted from any section. Each question carries 14 marks.

SECTION A

Q1) (a) An aircraft pursues a straight course with constant velocity V and is being chased by a guided missile moving with constant speed 2V and fitted with a homing device to ensure that its motion is always directed at the target. Initially the missile is at right angles to the course of the aircraft and distance R from it. Find the polar equation of the missile's pursuit curve relative to the target, taking the course of the target as the initial line $\theta = 0$, and find the time taken by it to strike the target. (9)

(b) A rigid body has a spin ω and a particle a of the body has velocity $\vec{v}$. Show that every particle P of the body with velocity vector parallel to $\vec{\omega}$ lies on the line

$$\overrightarrow{AP} \equiv \overrightarrow{(\omega \times \vec{v})} / \omega^2 + \mu \vec{\omega}$$

μ being an arbitrary scalar. (5)

Q2) (a) Prove that $\vec{\omega} = \frac{1}{2}$ Curl $\vec{v}$ (5)

(b) Discuss velocity and acceleration components in cylindrical polar coordinates. (9)

Section B

Q3) (a) Show that necessary and sufficient condition for a field of force $\vec{F}$ to be conservative is that Curl $\vec{F} = 0$ (7)

(b) State and prove Principle of Conservation of Energy. (7)

Q4) A small stone of mass m is thrown vertically upwards with initial speed V. If the air resistance at speed v is mkv^2, where k is a positive constant, show that the stone returns to its starting point with speed

$$V\left[1+\frac{kV^2}{g}\right]^{-1/2}$$

If the stone has the same mass and initial speed but if the air resistance is mkv, Show that the stone returns to its starting point with speed U given by the equation (14)

$$g - kU = (g + Kv) \exp\{-k(U+V)/g\}$$

Section C

Q5 Discuss the motion of particle on a cycloid. (14)

Q6) (a) A particle P of mass m moving in a circle with radius 'a' and center O under a force $\mu m[r + 2a^3/r^2]$ directed towards O. If P is acted upon by an impulse tangential to the path and of magnitude $(3\mu m^2a^2)^{1/2}$. Show that the velocity of P is immediately doubled and that the greatest and least distances from O in the ensuing motion are a and 3a. (9)

(b) A particle is projected with velocity V from the cusp of a smooth inverted cycloid down the arc, show that time of reaching the vertex is

$$2\sqrt{\frac{a}{g}}\tan^{-1}\frac{\sqrt{4ag}}{V} \qquad (5)$$

Section D

Q7) (a) A square of side 'a' has particles of mass m, 2m, 3m, 4m at its vertices. Show that the principal moments of inertia at the centre of square are $2ma^2$, $3ma^2$, $5ma^2$ .Also find the directions of principal axes. (7)

(b). Find the equation of momental ellipsoid of the rectangular block referred to the axes OX, OY, OZ where O is a corner of the rectangular block. (7)

Q8) (a) Define Principal Axes. Show that at each point of a body there are three principal axes mutually at right angles to each other such that the product of inertia about them taken two at a time is zero. (9)

(b) State and prove the theorem of parallel axes for moments of inertia. (5)

Exam Code: 225701 **Paper Code: 1221**

Programme: Master of Science (Mathematics)

Semester – I

Course Title: Differential Equations

Course Code: MMSL-1335

Time Allowed: 3 Hours **Max. Marks: 70**

Note: Attempt five questions in all, selecting at least one from each section. Fifth question may be attempted from any section. Each question carries 14 marks.

Section-A

1. State and prove Sturm's Separation theorem. **(14)**

2. (a) If $f_1(x), f_2(x)$ are two linear independent solution of $a_0y'' + a_1y' + a_2y = 0$ where $a_0 \neq 0 \;\forall\; x \in (a, b)$ then prove that $W(y_1, y_2, x) = W(y_1, y_2, x_0)\, e^{-\int \frac{a_1 t}{a_0 t} dt}$ where x_0 is point in (a,b) **(7)**

   (b) Find the third approximation of $\frac{dy}{dx} = 2x + z,\ \frac{dz}{dx} = 3xy + x^2 z$ where y=2, z=0 at x=0 **(7)**

Section-B

3. (a) Prove that $L\left\{\frac{Cos\,at - Cos\,bt}{t}\right\} = \frac{1}{2}\log\frac{s^2+b^2}{s^2+a^2}$ **(7)**

   (b) Evaluate $L^{-1}\left\{\frac{3s+7}{s^2-2s-3}\right\}$ **(7)**

4. (a) If $L\{F(t)\} = f(s)$ *then prove that*

$$L\{t^n F(t)\} = (-1)^n \frac{d^n}{ds^n} f(s) \; ; n = 1,2,3 \ldots..$$ **(7)**

(b) Solve the following differential equation using Laplace Transform $(D^2 + 4D + 8)y = 0, \quad given\ that$ $y(0) = 2, y'(0) = 2$ **(7)**

Section-C

5. (a) If $F(x) = Sin\ nx\ , n \in Z$. Show that $f_c(m) = \begin{cases} 0, \ if\ (n-m) is\ even \\ \frac{2n}{n^2-m^2}, \ if\ (n-m) is\ odd \end{cases}$ where m= 1,2,3…. **(7)**

(b) Show that the Fourier transform of

$F(x) = e^{-\frac{x^2}{2}}\ is\ \sqrt{2\pi}\ e^{-\frac{p^2}{2}}$ **(7)**

6. (a) State and prove Convolution theorem. **(7)**

(b) If $f_c(p) = \frac{1}{1+p^2}\ then\ find\ F_c^{-1}\left\{\frac{1}{1+p^2}\right\}$ **(7)**

Section-D

7. (a) Show that $x\, J_n'(x) = n\, J_n(x) - x\, J_{n+1}(x)$ **(7)**

(b) State and prove orthogonality for $P_n(x)$ **(7)**

8. (a) Prove that $e^{2tx-t^2} = \sum_{n=0}^{\infty} \frac{t^n}{n!} H_n(x)$ **(7)**

(b) Prove that

$$(n+1)L_{n+1}(x) = (2n+1-x)L_n(x) - n\, L_{n-1}(x)$$ **(7)**