Exam Code: 221002 **Paper Code: 2225**

Programme: Master of Science (Mathematics) Semester-II

Course Title: Real Analysis-II

Course Code: MMSL-2331

Time Allowed: 3 Hours **Max Marks: 80**

1) **Paper consists of Eight questions of equal marks (16 mark each)**
2) **Attempt Five questions in all by selecting atleast One question from each of Four section. FIFTH question may be attempted from any section.**

SECTION-A

1. (a) Let $\{f_n\}$ be a sequence of real valued functions defined on the closed and bounded interval $[a, b]$ and let $f_n \in R[a, b]$, for $n = 1, 2, 3, \ldots$. If f_n converges uniformly to the function f on $[a, b]$, then prove that $f \in R[a, b]$ and

$$\int_a^b f(x)dx = \lim_{n\to\infty} \int_a^b f_n(x)dx$$

(10)

(b) Examine for term by term integration the series, the sum of whose first n terms is

$$n^2x(1-x)^n \qquad (0 \le x \le 1)$$

(6)

2. State and prove Stone-Weierstrass Theorem. (16)

SECTION-B

3. (a) Prove that the outer measure of an interval is its length. (10)

(b) Find the length of the set $\bigcup_{k=1}^{\infty} \left\{x : \frac{1}{k+1} \le x < \frac{1}{k}\right\}$. (6)

4. (a) Let $\{E_i\}$ be an infinite increasing sequence of measurable sets, that is, a sequence with $E_{i+1} \supset E_i$ for each $i \in \mathbb{N}$. Then prove that

$$m\left(\bigcup_{i=1}^{\infty} E_i\right) = \lim_{n\to\infty} m(E_n)$$

(8)

(b) Let f be a function defined on a measurable set E. Then prove that f is measurable if and only if, for any open set G in $\mathbb{R}$, the inverse image $f^{-1}(G)$ is a measurable set. (8)

SECTION-C

5. A bounded function f defined on a measurable set E of finite measure is Lebesgue integrable if and only if f is measurable. (16)

6. (a) Let f be a bounded fucntion defined on $[a, b]$, If f is Reimann integrable over $[a, b]$, then it is Lebesgue integrable and

$$\Re \int_a^b f(x)dx = \int_a^b f(x)dx$$

(8)

(b) **(Monotone Convergence Theorem)** Let $\{f_n\}$ be an increasing sequence of non-negative measurable functions, and let $f = \lim_{n\to\infty} f_n$. Then prove that

$$\int f = \lim_{n\to\infty} \int f_n$$

(8)

SECTION-D

7. (a) If f is any function on an interval I, then prove that $\overline{D}f$ and $\underline{D}f$ are measurable functions. (8)

(b) Prove that the union of any collection of intervals is a measurable set. (8)

8. Let f be an increasing real-valued function defined on $[a, b]$. Then prove that f is differentiable a.e.(*almost everywhere*) and the derivative f' is measurable. Furthermore,

$$\int_a^b f'(x)dx \leq f(b) - f(a)$$

(16)

Exam Code: 221002 **Paper Code: 2226**

(20)

Programme: Master of Science (Mathematics)
Semester-II

Course Title: Tensors and Differential Geometry

Course Code: MMSL-2332

Time Allowed: 3 Hours **Max Marks: 80**

Attempt five questions, selecting at least one question from each section. The fifth question may be attempted from any section. Each question carries 16 marks.

Section-A

1. (a) Show that inner product of the tensors A_r^p and B_t^{qs} is a tensor of rank three (4)

   (b) If a_{ij} is a second rank covariant symmetric tensor and $|a_{ij}|$ = a then show that $\sqrt{a}$ is a scalar density. (4)

   (c) Find the metric of a Euclidean space referred to cylindrical co-ordinates. (8)

2. (a) Show that the covariant derivative of a contra variant vector is a mixed tensor of rank two. (8)

   (b) Show that g_{ij} is a second rank covariant symmetric tensor. (8)

Section-B

3. (a) Find the radii of curvature and torsion for the curve $x = 3u, y = 3u^2, z = 2u^3$. (8)

   (b) Prove that for all helices curvature bears a constant ratio with torsion. (8)

4. (a) Show that a necessary and sufficient condition for a curve lies on a sphere is that $\frac{f}{\sigma} + \frac{d}{ds}\left(\frac{p^1}{z}\right)$=0 at every point on the curve. (8)

   (b) Obtain the curvature and torsion of spherical indicatrix of the tangent. (8)

Section-C

5. (a) Prove that necessary and sufficient condition for a surface to be developable surface is that its Gaussian curvature should be zero. (8)

   (b) State and prove Beltranil and Enneper theorem.(8)

6. (a) Obtain the Manardi-codazzi equations in their usual form. (8)

   (b) Find the differential equation of lines of curvature of the helicoid $x = u\cos v, y = u\sin v \, and$ $z = f(u) + cv$. (8)

Section-D

7. (a) State and prove necessary and sufficient condition for a curve on a surface to be geodesic. (8)

   (b) Prove that two geodesic at right angles have their torsions equal in magnitude but opposite in sign. (5)

   (c) Prove that the curvature of a geodesic relative to itself is zero. (3)

8. (a) State and prove Gauss-Bonnet theorem. (8)

   (b) State and prove Joachimsthal theorem. (8)

Exam Code: 221002
(20)

Paper Code: 2227

Programme: Master of Science (Mathematics)
Semester-II

Course Title: Algebra-II

Course Code: MMSL-2333

Time Allowed: 3 Hours **Max Marks: 80**

Attempt five questions selecting at least one question from each section. The fifth question may be attempted from any section. Each question carries 16 marks.

Section-A

1. (a) Let R be commutative ring with unity then R[x] is P.I.D iff R is a field. 12

   (b) Is $\frac{\mathbb{Q}[x]}{<x^2-5x=6>}$ is a field? Why? Here $\mathbb{Q}$ is a field of rationals. 04

2. (a) Every Euclidean domain is Principal Ideal domain.06

   (b) Let D be a U.F.D. and $f(x) \in D[x]$ be anirreducible element of D[x] then either $f(x)$ is irreducible element of D or $f(x)$irreducible primitive polynomial over F, where F is field of quotient of D. 10

Section-B

3. (a) If E is finite extension of finite field F then E is simple extension of F. 08

(b) Show by an example that finitely generated field extension may not be finite extension. 04

(c) Let K be field extension of F and $a \in k$ is algebraic over F of an odd degree then F (a) = F (a^2) 04

4. (a) If F is finite field of characteristic þ, show that each element 'a' of F has a unique pth root in F. 05

(b) Let α be a root of $x^{þ} - x - 1$ over a field F of characteristic p. Show that F(α) is a separable extension of F. 04

(c) Let K be a field extension of F, $a \in k$ be algebraic over F then F[a] is field. 07

Section-C

5. (a) Prove or disprove if F⊆E⊆K be chain of fields such that E is normal extension of F and K is normal extension of E then K is normal extension of F. 06

(b) Show that Galois group of $x^4 - 2 \in \mathbb{Q}[x]$ is the octic group. 10

6. (a) If $f(x) \in F[x]$ has r distinct roots in its splitting field E over F then the Galois group G(E/F) of $f(x)$ is a subgroup of symmetric group S_r. 10

(b) Show that the polynomial $x^7 - 10x^5 + 15x + 5$ is not solvable by radicals over $\mathbb{Q}$, field of rationals. 06

Section-D

7. (a) Prove that over a commutative ring, any two basis of a finitely generated free module have same number of elements. 10

(b) Let M be cyclic R-module that is M=Rm for same m∈M then $M \cong \frac{R}{I}$ for some left ideal I of R. 06

8. (a) Let R be a ring with unity. Prove that in an R-module R a left ideal A is direct summand of R iff A=Re for same idempotent e of R. 6

(b) Prove or disprove that every submodule of an R-module is direct summand. 05

(c) Show by an example that submodule of finitely generated module may not be finitely generated. 05

Exam Code: 221002 **Paper Code: 2228**
(20)

Programme: Master of Science (Mathematics) Semester-II

Course Title: Mechanics-II

Course Code: MMSL-2334

Time Allowed: 3 Hours **Max Marks: 80**

Attempt a total of five questions selecting at least one question from each section. The fifth question may be attempted from any section. Each question carries 16 marks.

Section A

1. (a) State and prove the principle of conservation of angular momentum. [8]
   (b) Establish that the linear momentum is constant for a system of particles having no resultant force. [4]
   (c) Derive an expression for $K.E.$ of a rigid body moving in two dimensions. [4]
2. (a) A uniform rod of mass M and length $2a$ lies at rest on a smooth horizontal table. An impulse J is applied at A in the plane of the table and perpendicular to the rod. Determine the velocity of the centroid and the angular velocity of the rod. [4]
   (b) Show that the rate of change of vector angular momentum of a system of particles moving generally in space is equal to the moments of the external forces acting on the system.(External and internal forces). [6]
   (c) Two particles of masses m_1 and m_2 at A and B are connected by a rigid rod AB lying on a smooth horizontal table. If an impulse I is applied at A in the plane of the table and perpendicular to AB. Find the initial velocities of A and B. [6]

Section B

3. (a) Find the kinetic energy of a rigid body rotating about a fixed point with velocity $\vec{\omega}$. [8]
   (b) Prove that a rigid body motion about a fixed point under no force is equivalent to rolling of one cone on another. [8]
4. (a) Show that the product of inertia with respect to principal axes vanish. [10]
   (b) Show that vector angular momentum about a fixed point of a particle moving under no forces is constant. [6]

Section C

5. (a) Two uniform rods AB, AC each of mass m and length $2a$, are smoothly hinged together at A and move on a horizontal plane. At time t the mass-centre of the rods is at the point (ξ, η) referred to fixed perpendicular axes Ox, Oy in the plane and the rods make angle $\theta \pm \phi$ with Ox. Prove that the kinetic energy of the system is $m[\dot{\xi}^2 + \dot{\eta}^2 + \left(\frac{1}{3} + \sin^2\phi\right) a^2 \dot{\theta}^2 + \left(\frac{1}{3} + \cos^2\phi\right) a^2 \dot{\phi}^2]$. [8]
   (b) Derive Lagrange's equations for Impulsive Forces. [8]
6. (a) Determine Lagrange's equations of the motion of a planet of mass m orbiting round the sun under inverse square law of attraction. [6]

(b) A horizontal circular wire has radius R, centre C and is free to rotate about a vertical axis through a point O in its plane distant d from C. The wire carries a smooth particle P and $\angle OCP = \theta$ at time t. If ω is the angular velocity of the wire, show that $R\ddot{\theta} + \dot{\omega}(R - d\cos\theta) = d\omega^2 \sin\theta$. [6]

(c) Define

(i) Generalised coordinates of a dynamical system

(ii) Impulsive virtual work function.

(iii) Generalised forces.

(iv) Holonomic System. [4]

Section D

7. (a) Find the extremals of the functional $\int_0^1 (xy + y^2 - 2y^2 y')dx$; $y(0) = 1$ and $y(1) = 2$ [3]

(b) Show that the geodesics on a sphere of radius a are its great circles. [8]

(c) Find approximately the smallest eigen value λ of $y'' + \lambda y = 0$; $y(0) = y(1) = 0$ [5]

8. (a) Find the equation of motion of one dimensional harmonic oscillator using hamilton's principle. [8]

(b) Distinguish between Hamilton's Principle and the Principle of Least Squares. [4]

(c) Solve the boundary value problem $y'' - y + x = 0\ (0 \le x \le 1)$, $y(0) = y(1) = 0$ by Rayleigh Ritz method. [4]

Exam Code: 221002
(20)

Paper Code: 2229

Programme: Master of Science (Mathematics) Semester-II

Course Title: Differential and Integral Equations

Course Code: MMSL-2335

Time Allowed: 3 Hours **Max Marks: 80**

Note: Attempt five questions in all, selecting at least one question from each section. The fifth question may be attempted from any section. Each question carries 16 marks.

Section-A

1(a) Find the general integral of the equation $(x-y)p+(y-x-z)q=z$ and the particular solution through the circle $z=1, x^2+y^2=1$

(b) Show that the equations $xp-yq=x, x^2p+q=xz$ are compatible and find their solution. [8,8]

2(a) Show how to solve, by JACOBI METHOD, a partial differential equation of the type $f(x, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial z})=g(y, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z})$ and illustrate the method by finding a complete integral of the equation $2x^2y\left(\frac{\partial u}{\partial x}\right)^2\frac{\partial u}{\partial z}=x^2\frac{\partial u}{\partial y}+2y\left(\frac{\partial u}{\partial x}\right)^2$

(b) Solve: $r+s-6t=y\cos x$ [10,6]

Section-B

3 (a) Reduce $x\frac{\partial^2 z}{\partial x^2}+2\frac{\partial^2 z}{\partial x\partial y}+\frac{\partial^2 z}{\partial y^2}=0$ to canonical form and hence solve it.

(b) Obtain the solution of $\frac{\partial^2 z}{\partial x\partial y}=\frac{1}{x+y}$ such that $z=0, p=\frac{2y}{x+y}$ $on\, y=x$. [8,8]

4(a) Find the solution of Laplace's equation in two dimensions by the method of Separation of Variables.

(b)Solve the Wave equation $r=t$ by MONGE'S METHOD. [8,8]

Section-C

5. Explain the method of successive substitutions for the solution of Volterra Integral Equation of second kind. [16]

6(a) Form an Integral equation corresponding to the following differential equation with the given initial conditions $y''+y=cosx, \quad y(0)=0, y'(0)=1$

(b) With the help of Resolvent kernel, find the solution of

$$F(x)=e^{x^2}+\int_0^x e^{x^2-\eta^2}F(\eta)d\eta$$ [8,8]

Section-D

7(a) Solve the Fredholm equation

$$u(x)=e^x-\frac{e-1}{2}+\frac{1}{2}\int_0^1 u(t)dt$$

(b) Compute $D(\lambda)$ for the Integral equation

$$y(x)=f(x)+\lambda\int_0^1(x+t)\,y(t)dt$$ [8,8]

8. Show that the solution $F(x)=G(x)+\lambda\int_a^b R(x,\eta;\lambda)G(\eta)d\eta$ of the non-homogeneous Fredholm's integral equation of second kind is unique provided $D(\lambda)\neq 0$ [16]