

**Exam Code: 508401**

**Paper Code: 1199**

**Programme: Master of Science (Mathematics) (FYIP)**

**Semester – I**

**Course Title: Calculus**

**Course Code: FMAL-1333**

**Time Allowed: 3 Hours**

**Maximum Marks: 70**

**Note:** Attempt five questions in all, selecting atleast one question from each section. Fifth question may be attempted from any section. Each question carries 14 marks.

**Section-A**

1. (a) Using definition of limit, prove that  $\lim_{x \rightarrow 2} (3x - 5) = 1$ . (04)  
(b) Find the domain of the continuity of  $f(x) = [x]$ , where  $[.]$  denotes the greatest integer function. (05)  
(c) Evaluate  $\lim_{x \rightarrow 0} (\cot^2 x - \frac{1}{x^2})$  (05)
2. (a) Evaluate  $\lim_{x \rightarrow 0} (\frac{1}{x})^{\tan x}$ . (05)  
(b) If  $y = (\sin^{-1} x)^2$ , find  $y_n(0)$ . (05)  
(c) Find the nth order derivative of  $e^{3x} \sin^2 x \cos^3 x$ . (04)

**Section-B**

3. (a) Find the intervals of increase and decrease for the function  $f(x) = 2x^3 - 3x^2 + 6x - 7$ . (04)  
(b) Show that the function  $f(x) = |x|$  is differentiable for all real numbers except at  $x=0$ . (05)  
(c) Prove that among all the rectangles of given perimeter, the square has largest area. (05)

4. (a) Calculate the approximate value of  $\sqrt{26}$  to three decimal places by Taylor's expansion. (07)

(b) State and prove Maclaurin's Theorem with Lagrange's form of remainder. (07)

### Section-C

5. (a) Find the values of  $x$  for which the curve  $y = x^4 - 6x^3 + 12x^2 + 5x + 7$  is Concave upwards or downwards. Also determine the points of inflexion. (06)

(b) Determine  $a$  and  $b$  so that the curve  $y = ax^3 + 3bx^2$  has a point of inflexion at  $(-1, 2)$ . (06)

(c) Define horizontal asymptote. (02)

6. (a) Find all the asymptotes of the curve  
 $x^3 + 2x^2y - xy^2 - 2y^3 - y^2 + xy - 1 = 0$  (07)

(b) Find the equation of cubic curve which has the same asymptote as the curve  
 $x^3 - 6x^2y + 11xy^2 - 6y^3 + 4x + 5y + 7 = 0$  and which passes through the points  $(0, 0)$ ,  $(-2, 0)$  and  $(0, -2)$ . (07)

### Section-D

7. (a) State and prove Fundamental Theorem of Integral Calculus. (07)

(b) If  $U_n = \int_0^{\frac{\pi}{4}} \tan^n x \, dx$   $n > 1$ , then show that  $U_n + U_{n-2} = \frac{1}{n-1}$ . Deduce the value of  $U_5$ . (07)

8. (a) Prove that  $\int_a^b f(x) \, dx = \int_a^c f(x) \, dx + \int_c^b f(x) \, dx$ . (02)

(b) Find the length of arc of parabola  $y^2 - 4y + 2x = 0$  which lies in the first quadrant. (06)

(c) Find the area above the  $x$ -axis included between the curve  $y^2 = 2ax - x^2$  and  $y^2 = ax$ . (06)

**Exam Code: 508401**

**Paper Code: 1200**

**Programme: Master of Science (Mathematics) (FYIP)**

**Semester-I**

**Course Title: Theory of Equations**

**Course Code: FMAL-1334**

**Time Allowed: 3 Hours**

**Max. Marks: 70**

Note: Attempt five questions in all, selecting at least one from each section. Fifth question may be attempted from any section. Each question carries 14 marks.

**Section-A**

1(a). Write down the general properties of polynomials. (7)

1(b). Find the quotient and the remainder when  $f(x) = x^3 + x - 1$  is divided by  $g(x) = 2x - 1$  over the set of rational numbers. (7)

2(a). Trace the polynomial  $10x^3 - 17x^2 + x + 6$ . (7)

2(b). If -2 is a root of  $x^4 - (2a + 3)x^2 - 2(a - 1)x + 12 = 0$ , check that it is a repeated root, if so then find its multiplicity. Also solve the equation. (7)

**Section-B**

3(a). Show that the equation  $2x^7 + 3x^4 + 3x + k = 0$  has at least four imaginary roots for all values of k. (7)

3(b). Find the rational roots of the equation  $2x^3 - 11x^2 - 7x + 6 = 0$ . (7)

4(a). Solve the equation  $32x^3 - 48x^2 + 22x - 3 = 0$ , the roots being in A.P. (7)

4(b). If  $\alpha, \beta, \gamma$  are roots of  $x^3 - px^2 + qx - r = 0$ , find the value of  $\sum(\frac{\alpha}{\beta} + \frac{\beta}{\alpha})$ . (7)

### Section-C

5(a). Explain Ferrari's method of solving a biquadratic equation. (7)

5(b). If  $\alpha, \beta, \gamma$  are roots of the equation  $x^3 + 2x + 6 = 0$ , prove that  $S_7 = 2(S_4 - S_6)$ . (7)

6(a). Find the equation of the squared differences of the roots of the cubic  $x^3 - 9x^2 + 23x - 15 = 0$ . (7)

6(b). Solve the reciprocal equation  $x^4 - 3x^3 + 4x^2 - 3x + 1 = 0$ . (7)

### Section-D

7(a). Find the upper and lower limit of the positive roots of the equation  $2x^5 - 7x^4 - 5x^3 + 6x^2 + 3x - 10 = 0$ . (7)

7(b). Separate the real roots of the equation  $x^4 - 6x^2 + 8x - 8 = 0$ . (7)

8. State and Prove Newton's theorem on the sum of powers of roots. (14)

Exam Code: 508401

Paper Code: 1202

Master of Science (Mathematics) (FYIP) Semester-I

Course Title: Mechanics-I

Course Code: FMAM-1395

Time: 3 Hours

Max. Marks: 40

Note: Attempt five questions, selecting one question from each section. The fifth question can be attempted from any section. Each question carries 8 marks. Students can use Non-Scientific calculators or logarithmic tables.

#### SECTION A

1. What are spherical polar coordinates? Define radial distance, azimuthal angle and zenith angle. Obtain their relation with Cartesian coordinates of a point. Also find expression for unit vectors of spherical polar coordinate system. (8)
2. (a) Define solid angle and find the solid angle subtended by the surface of sphere at its center. (4)  
(b) The cartesian coordinates of a particle vary with time as  $x = 2\sin 4t, y = 2\cos 4t, z = 4t$ , find velocity and acceleration of particle? (4)

#### Section B

3. Discuss elastic scattering of two particles in center of mass system and prove that the speeds of colliding particles remain unchanged but they move in opposite directions after collision. (8)
4. Discuss the rotation of angular momentum vector about a fixed axis for a rigid rotator. Hence prove that a torque is always required to produce this rotation. (8)

#### Section C

5. (a) Discuss the conditions for various shapes of orbits from a graph showing the variation of centrifugal potential energy, gravitational potential energy and effective potential energy with radial distance from center of force. (4)

(b) A body moving under action of a central force describes a path  $r = a \sin\theta$ . Find the force law. (4)

6. Derive the differential equation of orbit by reducing a two body problem into a single body problem. Also prove that the total energy and angular momentum are always conserved for motion under central force? (8)

#### Section D

7. Find expressions for coriolis force on particle moving in a non inertial rotating frame of reference. Discuss both the cases (i) when particle is falling freely under some acceleration "g" (ii) when particle is moving with some constant velocity on the surface of frame. (8)

8. What is Foucault pendulum? Prove that the plane of oscillation of the bob of Foucault's pendulum traces an elliptical path due to rotation of earth. Also find the time period of rotation at poles and equator. (8)

Exam Code: 508401

Paper Code: 1203

Programme: Master of Science (Mathematics) FYIP

Semester – I

Course Title: Programming Language –I

Course Code: FMAM-1136

Time Allowed: 3 Hours

Max. Marks: 40

NOTE: Candidates are required to attempt five questions, selecting at least one question from each section. The fifth question may be attempted from any section. Each question carries 08 marks.

**SECTION - A**

- Q.1 What do you mean by Program Development Life Cycle? Explain its main phases. (8)
- Q.2 What is the use of Decision Table in programming? Describe different components of a decision table. (8)

**SECTION - B**

- Q.3 What are Branching and Jumping Statements? Explain with syntax and suitable examples. (8)
- Q.4 What do you understand by an array? How do you declare and initialize an array in C? Provide examples for single-dimensional and multi-dimensional arrays. (8)

**SECTION - C**

- Q.5 What is the importance of Storage classes in C Language? Describe various types of storage classes. (8)
- Q.6 (a) Write a recursive C function to compute the nth Fibonacci number. (4)
- (b) Discuss standard library functions. (4)

**SECTION - D**

- Q.7 Explain File handling in C. Why is it important for data storage? Explain different modes to open a file in C. (8)
- Q.8 Write an algorithm to sort the elements of an array using quick sort. (8)