Exam Code: 508403 **Paper Code: 3205**
(20)

Programme: Master of Science (Mathematics) (FYIP) Semester-III

Course Title: Calculus-III

Course Code: FMAL-3331

Time Allowed: 3 Hours **Max Marks: 80**

Note : Candidates are required to attempt five questions, selecting at least one question from each section. The fifth question may be attempted from any section. Each question carries 16 marks.

Section A

1. (a) Show that the function $f(x, y) = \cos(x + y)$ is differentiable at $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$.at the origin. 8

(b) Find second order partial derivative of $\log(e^x + e^y)$. 8

2.(a) Use definition to show that $\lim_{(x,y)\to(1,2)} x^2 + y^2 = 5$ 8

(b) State and prove Schwarz's theorem. 8

Section B

3. (a) Find the directional derivative of $\phi(x, y, z) = x^2yz + 4xz^2$ at the point $(1, -2, 1)$ in the direction $2\hat{\imath} - \hat{\jmath} - 2\hat{k}$ 8

(b) Find $J_f(x, y)$, when $f(x, y) = (x^2, y)$ 8

4. (a) If λ, μ, ν are the roots of the equation

$\frac{x}{a+k} + \frac{y}{b+k} + \frac{z}{c+k} = 1$ prove that $\frac{\partial(x,y,z)}{\partial(\lambda,\mu,\nu)} = -\frac{(\mu-\nu)(\nu-\lambda)(\lambda-\mu)}{(b-c)(c-a)(a-b)}$ 8

(b) If $x = r\cos\theta, y = r\sin\theta$, find $\frac{\partial(x,y)}{\partial(r,\theta)}$ 8

Section C

5 (a) Find the extreme value of the function 8

$$f(x,y) = 2x^4 - 3x^2y + y^2$$

(b) Show that of all the triangles inscribed in a circle, the one with the maximum area is equilateral. 8

6. (a) State and prove Taylor's theorem for function of two variables. 8

(b) Expand y^x up to second degree terms at (1,1). 8

Section D

7 (a) Evaluate $\int_0^{\pi}\int_0^{x} x \sin y \, dxdy$ 8

(b) Evaluate $\int_0^1\int_{4y}^{4}(e^{x^2})\,dxdy$ by changing the order of integration. 8

8 (a) Evaluate $\int\int(4 - x^2 - y^2)\,dxdy$ over the region bounded by the semi-circle $x^2 + y^2 - 2x = 0$ and the co-ordinate axes lying in the first quadrant. 8

(b) A square ABCD is divided into two parts by joining A to E, the middle point of BC. Prove that the line joining the centre of gravity of ΔABE to that of a quadrilateral AECD is perpendicular to AE. 8

Exam Code: 508403 **Paper Code: 3206**
(20)

Programme: Master of Science (Mathematics) (FYIP) Semester-III

Course Title: Ordinary Differential Equations and Special Functions

Course Code: FMAL-3332

Time Allowed: 3 Hours **Max Marks: 80**

Note : Attempt FIVE question in all, selecting at least ONE question from each section. The fifth question may be attempted from any section. Each question carries equal marks. (16 each)

SECTION-A

1(a) Solve the differential equation $(\cos x \cos y - \cot x)dx - (\sin x \sin y)dy = 0$ 8

(b) Solve $y = 2p + 3p^2$, *where* $p = \frac{dy}{dx}$ 8

2(a) Find orthogonal trajectories of family of ellipses $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. 8

(b) Solve the differential equation $(D^2 - 4D - 4)y = 8(e^{2x} + \sin 2x)$, *where* $D = \frac{d}{dx}$. 8

SECTION-B

3(a) Solve the differential equation $(x^3D^3 + 2x^2D^2 + 2)y = 10\left(x + \frac{1}{x}\right)$, *where* $D = \frac{d}{dx}$. 8

(b) Using the method of variation of parameters, solve the differential equation $(D^2 + a^2)y = \sin ax$, *where* $D = \frac{d}{dx}$. 8

4. Solve in series the Bessel's equation of order n;

$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0$, where $2n$ is non-integer. 16

SECTION-C

5(a) Prove that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}}\cos x$ *and* $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\sin x$ 8

(b) Prove that $\frac{d}{dx}\{x^{-n}J_n(x)\} = -x^{-n}J_{n+1}(x)$. 8

6(a) Show that $x\sin x = 2(2^2J_2 - 4^2J_4 + 6^2J_6 - ....)$. 8

(b) Use the generating function or otherwise to show that $J_n(x) = (-1)^n J_n(x)$ 8

SECTION-D

7. Prove that $\int_{-1}^{1} P_n(x)P_m(x)\,dx = \begin{cases} 0 & ,if\ m \neq n \\ \frac{2}{2n+1} & ,if\ m = n \end{cases}$ 16

8(a) Using Rodrigue's formula, find values of $P_0(x)$, $P_1(x)$, $P_2(x)$, and $P_3(x)$. 8

(b) If n is a positive integer, then prove that $P_n(x) = \frac{1}{\pi}\int_0^{\pi}\left[x \pm \sqrt{(x^2-1)}\cos\phi\right]^n d\phi$ 8

Exam Code: 508403 (20)

Paper Code: 3207

Programme: Master of Science (Mathematics) (FYIP) Semester-III

Course Title: Probability Theory

Course Code: FMAL-3333

Time Allowed: 3 Hours **Max Marks: 80**

Note : Attempt FIVE question in all, selecting at least ONE question from each section. Fifth question can be attempted from any section. Each question carries equal (16) marks. Students can use only Non-Programmable & Non-Storage Type calculator and statistical Tables.

Section A

Q1. (a) What is Dispersion? Why is it measured? Give its various measures. [8]

(b) Based upon moments comment upon the nature of Skewness and Kurtosis of the given data: [8]

Marks	5-15	15-25	25-35	35-45	45-55
Students	1	3	5	7	4

Q2. (a) Bowl A contains 3 red and 7 blue chips and bowl B contains 8 red and 2 blue chips. Dice is rolled and bowl A is selected if five or six shows up; otherwise, bowl B is selected. Find the conditional probability of choosing bowl A, given that a red chip is drawn. [8]

(b) A speaks the truth in 60% and B in 75% of the cases. In what percentage of cases are they likely to contradict each other in stating the same fact? [8]

Section B

Q3 (a) The joint density function of two random variables X and Y is given by

$$f(x,y) = \begin{cases} k(x+y); & x = 1,2,3;\ y = 1,2 \\ 0; & elsewhere \end{cases}$$

Find (i) k (ii) $P(x < 2 \cap y < 3)$ (iii) $P(x + y < 4)$ (iv) $P\{x < 2 |\ y < 2\}$. [8]

(b) Let X has probability density function $f(x) = \frac{1}{\pi(1+x^2)}, -\infty < x < \infty.$

(i) Find the probability that X^2 lies between $\frac{1}{3}$ and 1.

(ii) Find the distribution function of X. [8]

Q.4 (a) The joint density function of two continuous random variables X and Y is given by

$$f(x,y) = \begin{cases} Ae^{-x-y}; & 0 \le x \le y,\ 0 \le y < \infty \\ 0; & elsewhere \end{cases}$$

(i) Find A (ii) Find marginal densities of X and Y (iii) Examine if X and Y are independent (iv) Find the conditional density function of Y given X=2. [8]

(b) Let X and Y be two independent and identically distributed random variables with common pdf as $f(x) = 1, 0 < x < 1$. Find the pdf of $W = \frac{X}{Y}$. [8]

Section C

Q5. The joint density function of two random variables X and Y is given by

$$f(x,y) = \begin{cases} \frac{6-x-y}{8}; & 0 \le x \le 2,\ 2 \le y \le 4 \\ 0; & elsewhere \end{cases}$$

Find (i) $E(X)$ (ii) $E(Y)$ (iii) $E(XY)$ (iv) $E(X^2)$

(v) $E(Y^2)$ (vi) $V(X)$ (vii) $V(Y)$ (viii) $Cov(X,Y)$. [16]

Q6. (a) For a random variable X, taking the value x with the probability $\frac{1}{2\sqrt{x}}$, $0 < x < 1$, find first three moments about the origin using moment generating function. [8]

(b) The density of random variable Y is $f(y) = e^{-y}$, $y > 0$ and zero otherwise. Using Chebychev's inequality find the lower bound to $P[-1 \le Y \le 3]$ and compare it with exact probability. [8]

Section D

Q7. (a) Find mean and variance of Binomial distribution with parameter n *and* p. [8]

(b) Show that the Geovietric Distributions has lack of memory property [8]

Q8. (a) Find the moment generating function of standard binomial variate and obtain its limiting form as $n \to \infty$. Also interpret the result. [8]

(b) Define Gamma distribution. Obtain its moment generating function. Hence, find its mean and variance. [8]

Exam Code: 508403 Paper Code: 3208

(20)

Programme: Master of Science (Mathematics) (FYIP)

Semester-III

Course Title: Linear Algebra

Course Code: FMAL-3334

Time Allowed: 3 Hours **Max Marks: 80**

Note : Attempt FIVE question in all, selecting at least ONE question from each section. The fifth question may be attempted from any section. Each question carries equal marks. (16 each)

Section A

1. (a) Define Ring. Give an example of

   (i) Commutative ring without unity,

   (ii) Commutative ring with unity . 04

   (b) Determine the subset $W = \{(a_1, a_2, a_3): a_1 = a_2 + 3\}$ form a subspace of R^3 or not. 04

   (c) Prove that union of two subspaces of V is a subspace of V iff one of the subspaces is contained in the other. 08

2. (a). Check for linear independence of the vectors (2, 2, 1), (1, 3, 1) and (1, 2, 2) in R^3(R).Also write (3, 1, 5) as linear combination of these vectors. 08

   (b) Prove or give a counter example: if U_1, U_2 and W are subspaces of V such that $V = U_1 \oplus W$ and $V = U_2 \oplus W$ then $U_1 = U_2$ 08

Section B

3. (a) Any two bases of a finite dimensional vector space have the same number of elements. 10

(b)Let U be the subspace of R^5 defined by

$U=\{(x_1, x_2, x_3, x_4, x_5) \in R^5 : x_1 = 3x_2, x_3 = 7x_4\}$,Find basis and dimension of U 06

4. (a) Let U and W be two subspaces of R^4, where

U= {(a, b, c, d): b + c + d=0} and W={(a, b, c, d): a + b=0, c=2d}. Find basis and dimension of (i) U (ii) W (iii) U∩ W. Also find dimension of U+W. 07

(b) Let V be finitely generated vector space over F. Then any maximal linearly independent subset of V is a basis of V. 07

(c) Define Quotient space. 02

Section C

5. (a) Check the mapping $T: R^2 \rightarrow R^2$ defined as $T(x_1, x_2) = (x_2, 0)$ is linear transmation. 04

(b) Find a linear transformation $T: R^4 \rightarrow R^3$ whose range space is spanned by (0,1,-3) and (0,-3,4). 06

(c) Let V and W be finite dimensional vector spaces with dimV=dimW. Then a linear transformation T : V→W is one-one iff T is onto. 06

6. (a) Let B= $\{v_1, v_2, \ldots, v_n\}$ be a basis for vector space V(F).Let T:V→V be a linear transformation. Prove that for any vector v in V, [T;B][v;B]=[T(v);B]. 06

(b) Let T be a linear operator on R^2 defined by T(x, y) = (3x-4y,x+5y). Find matrix of T relative to the basis B={(1,3), (3,4)}. 05

(c) If matrix of linear operator T on R^3 relative to usual basis is

$\begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$, then find the matrix of T relative to the basis 05

$\mathcal{B}$={(1,2,2), (1,1,2), (1,2,1)}.

Section-D

7. (a) Give definition of elementary matrix and also an example of it. 04

(b). Elementary row and column operations on a matrix are rank preserving. 12

8. (a) If A is an m*n ordered matrix of rank r over some field F, then there exist suitable invertible matrices P and Q such that PAQ=$\begin{bmatrix} I_r & O \\ O & O \end{bmatrix}$, where I_r is the identity matrix of order r. 10

(b) Determine whether the following system of linear equations has a solution:

x+ 2y- z =1

2x+ y+ 2z =3 06

x- 4y+7z =4.

Exam Code: 508403 Paper Code: 3209
(20)

Programme: Master of Science (Mathematics) (FYIP)
Semester-III

Course Title: Python Programming

Course Code: FMAM-3135

Time Allowed: 3 Hours **Max Marks: 50**

Note: Candidates are required to attempt five questions, selecting at least one question from each section. The fifth question may be attempted from any section. Each question carries 10 marks.

Attempt any Five Questions

Section — I

1. Discuss List and Tuples in details. What are various operations/functions that can performed on List and Tuples. 10
2. What do you mean by Strings and Dictionaries? What are various operations that can be applied on Strings and Dictionaries? 10

Section — II

3. Explain NumPY module in Python. How Matrix addition, subtraction and Multiplication can be achieved in Python using NumPY? 10
4. Write a Program in Python to Print Prime numbers upto 100. 10

Section — III

5. Explain the concepts of functions in python. Also explain lambda function and recursion with programming examples. 10
6. Explain functions to calculate Arithmetic Mean, Logarithmic functions and Trigonometric functions using math module in Python. 10

Section — IV

7. Explain how graphs can be drawn in Python? 10
8. Explain the concept of files in Python. What are various modes to open a file? 10