Exam Code: 221003 **Paper Code: 3218**
(20)

Programme: Master of Science (Mathematics) Semester-III

Course Title: Functional Analysis-I

Course Code: MMSL-3331

Time Allowed: 3 Hours **Max Marks: 80**

Note : Attempt five questions in all selecting at least one question from each section. Fifth question can be attempted from any section. Each question carries equal (16 each) marks

Section -A

Q-1 (a) Define summability in a normed linear space and Prove that a normed linear space N is a Banach Space iff every absolutely summable series in N is summable.

(b) State and prove Minkowski's inequality for finite form [8,8]

Q-2 (a) State and prove Riesz Holder inequality

(b)Let X and Y are normed linear spaces over field K and T:X→Y be a linear onto operator. Then T^{-1}exists and is bounded linear operator iff ∃ constant K> 0 such that $\|Tx\| \geq K\|x\|\ \forall\ x \epsilon X$ [8,8]

Section B

Q-3 (a)Define conjugate space of normed linear space and derive dual space of the normed linear space c_o

(b) Prove that any two n-dimensional normed spaces over the same field are topologically isomorphic [8,8]

Q-4 (a)Let M be a closed linear subspace of a normed linear space N and y is a vector not in M . Then there exist a functional F in N such that F(M)={0} and F(y)≠0. Also define Natural Embedding of N in N^{**} and show that J: N→ N^{**} defined as J(x)=F_x is an isometric isomorphism of N into in N^{**}

(b) Give an example of unbounded linear functional. Justify your answer [8,8]

Section -C

Q-5 (a) State and prove closed graph theorem.

(b) Define conjugate operator of T define on a normed linear space and prove that the conjugate operator is continuous map. [8,8]

Q-6 (a) Define Projection on a Banach Space .Let P be projection On a Banach Space B and let M and N be its range and Null Space respectively. Then M and N are closed linear manifolds of B such that B is the direct sum of M and N.

(b)Let T be a bounded linear operator on a normed space X. Then, the adjoint $T^*:X^* \to X^*$ defined by

(T^*g) (x) = g(Tx), g$\epsilon X^*$and $x\epsilon X$ is a bounded linear operator [8,8]

Section-D

Q-7 (a) A Hilbert Space H is finite Dimensional if and only if every complete orthonormal set is a basis of H

(b) For the special Hilbert space l_2^n,use Cauchy's inequality to prove Schwarz's inequality. [8,8]

Q 8-(a) Define Inner Product Space and Prove that the linear space c^n is an inner product space

(b) State and prove Riesz Representation Theorem. [8,8]

Exam Code: 221003 **Paper Code: 3219**
(20)

Programme: Master of Science (Mathematics)
Semester-III

Course Title: Topology-I

Course Code: MMSL-3332

Time Allowed: 3 Hours **Max Marks: 80**

Note:- Attempt five questions in all selecting at least one question from each section. Fifth question can be attempted from any section. Each question carries equal marks(16 each)

Section-A

1. (a) Give characterization of topology in terms of interior operator. (8)

   (b) Prove that if a space is second countable then it has a countable dense subset. (8)

2. (a) Prove that Lindel of is not hereditary property. (8)

   (b) Give an example of two different basis for the usual topology on the plane R^2. (8)

Section-B

3. (a) Prove that union of a collection of connected subspaces of X that have a point in common is connected. (8)

   (b) Prove that intervals and only intervals are connected subsets of usual topological space. (8)

4. (a) Define Homeomorphism. If $f: R \rightarrow R$ be defined as $f(x) = x + 10$, then prove that f is open. Is f a homeomorphism? (8)

   (b) Can a sequence converge to more than one point in a space ? Justify. If x is a limit point of a subset of a metric space, prove that there exists a sequence of points of the subset converging to x. (8)

Section-C

5. (a) Prove that product of two regular spaces is a regular space. (8)

   (b) Prove that every T_3 space is T_2- space but converse need not be true. (8)

6. (a) Prove that the property of being a T_4 space is hereditary property. (8)

   (b) State and Prove Tietze-Extension theorem. (8)

Section-D

7. (a) Prove that product topology is the smallest topology on the product space for which the projection mappings are continuous. (8)

   (b) Prove that an infinite product of non trivial discrete spaces is not discrete. (8)

8. (a) Define Quotient map. Give an example of a quotient map which is not closed. (8)

   (b). If the Projection P of a space X onto quotient space $\frac{X}{R}$ is open and R is closed in X×X then $\frac{X}{R}$ is Hausdorff space. (8)

Exam Code: 221003 Paper Code: 3220
(20)

Programme: Master of Science (Mathematics) Semester-III

Course Title: Discrete Mathematics-I

Course Code: MMSL-3333 (Opt-I)

Time Allowed: 3 Hours **Max Marks: 80**

Note : Attempt five questions in all selecting at least one question from each section. Fifth question can be attempted from any section. Each question carries equal (16 each) mark.

SECTION-A

1. (a) Suppose that P_1, P_2, P_3 are distinct primes and that $n, k \in Z^+$ with $n = P_1^5, P_2^3, P_3^k$ Let A be the set of positive integer divisors of n and define the relation R on A by xRy if x(*exactly*) divides y. If there are 5880 ordered pairs in R, determine k and $|A|$. (4)

   (b) Define the relation R on the set Z by aRb if $a - b$ is a nonnegative even integer. Verify that R defines a partial order for Z. Is this partial order a total order? (4)

   (c) Let $A = \{a_1, a_2, a_3, \ldots, a_n\}$ be a set consisting of n elements. Then prove that total number of reflexive and symmetric relations in $A \times A$ are $2^{(1/2)(n^2-n)}$ on set A. (8)

2. (a) Let triangle ABC be equilateral, with $AB = 1$. Show that if we select ten points in the interior of this square, there must be at least two whose distance apart is less than $\frac{1}{3}$. (8)

   (b) For $k, n \in Z^+$, prove that if $kn + 1$ pigeons occupy n pigeonholes, then at least one pigeonhole has $k + 1$ or more pigeons roosting in it. (8)

SECTION-B

3. (a) Use truth tables to verify the logical equivalences $[p \to (q \vee r)] \Leftrightarrow [\neg r \to (p \to q)]$. (8)

   (b) If Rochelle gets the supervisor's position and works hard, then she'll gets a raise. If she gets the raise, then she'll buy a new car. She has not purchased a new car. Therefore either Rochelle did not get the supervisor's position or she did not work hard. Write the above argument in symbolic form and then establish the validity of argument or give a counter example to show that it is invalid. (8)

4. (a) Establish the validity of argument by using laws $[p \wedge (p \to q) \wedge (\neg q \vee r)] \to r$. (8)

   (b) For the universe of all people, Consider the open statements
   m(x): x is a mathematics professor c(x): x has studied calculus
   Establish the validity of argument

   All mathematics professors have studied calculus
   Leona is a mathematics professor
   Therefore Leona has studied calculus (8)

SECTION-C

5. (a) Let S be the set of all matrices

$$\begin{bmatrix} 0 & a \\ 0 & b \end{bmatrix}$$

with entries $a, b \in \mathbb{Z}$. Show that S is a semigroup under matrix multiplication and show that S has a right identity but no left identity. Also, determine all right identities. (8)

(b) Let X be a set and denote $P(X)$ the set of its subsets (its *power set*). Prove that $(P(X), \cup)$ and $(P(X), \cap)$ are commutative monoids with respective identities $\emptyset$ and X. Aso determine the invertible elements of the monoids. (8)

6. Let $S, T, U,$ and V be sets and let $X \subseteq S \times T$, $Y \subseteq T \times U$, and $Z \subseteq U \times V$ be subsets. Define

$$X * Y := \{(s,u) \in S \times U \mid \exists t \in T : (s,t) \in X \text{ and } (t,u) \in Y\} \subseteq S \times U$$

(a) Prove that $(X * Y) * Z = X * (Y * Z)$

(b) Let S be a set. Show that $(P(S \times S), *)$ is a monoid. Is it commutative? Here $P(.)$ denotes the power set.

(c) Find the invertible elements in the monoid of Question 6, part (b).

(16)

SECTION-D

7. (a) Find the generating function for the number of integer solutions to the equation $c_1 + c_2 + c_3 + c_4 = 20$ where $-3 \leq c_1, -3 \leq c_2, -5 \leq c_3 \leq 5$, and $0 \leq c_4$. (8)

(b) Find the coefficient of x^{83} in

$$f(x) = (x^5 + x^8 + x^{11} + x^{14} + x^{17})^{10}$$

(8)

8. (a) Find and solve a recurrence relation for the number of ways to park motorcycles and compact cars in a row of n spaces if each motorcycle requires one space and each compact car needs two. (All motorcycles are identical in appearance, as are the compact cars, and we want to use up all the n spaces.) (8)

(b) Solve the following recurrence relation by the method of generating functions

$$a_{n+2} - 2a_{n+1} + a_n = 2^n, \quad n \geq 0, \quad a_0 = 1,\ a_1 = 2$$ (8)

Exam Code: 221003 **Paper Code: 3221**
(20)

Programme: Master of Science (Mathematics) Semester-III

Course Title: Statistics-I

Course Code: MMSL-3334 (Opt-III)

Time Allowed: 3 Hours **Max Marks: 80**

Candidates are required to attempt five questions selecting least one question from each section. The fifth question may be attempted from any Section. All questions carry equal (16) marks. The students can use only Non-Programmable & Non-Storage Type Calculator and statistical tables.

SECTION A

1. (a) Prove that for any discrete frequency distribution, standard deviation is not less than mean deviation from mean.

(b) Define Skewness and Kurtosis. How can you broadly classify distributions according to these features?

2.a) For three non-mutually exclusive events A, B and C in a sample space, show that

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

b) Define Conditional Probability. State and prove Bayes Theorem of Probability

SECTION B

3.a) (i) If X is a random variable, prove that $|E(X)| \leq E(|X|)$

(ii) If X and Y are two random variables such that $X \leq Y$, prove that $E(X) \leq E(Y)$

b) Starting from the origin, unit steps are taken to the right with probability *p* and to the left with probability *q(=1-p)*. Assuming independent movements, find mean and variance of the distance moved from origin after *n* steps.

4.a) Define cumulative distribution function of the random variable and describe its properties.

b) Let (X, Y) be a two-dimensional non-negative continuous random variable having the joint density function:

$$f(x,y) = \begin{cases} 4xy\, e^{-(x^2+y^2)} & ; x \geq 0, y \geq 0 \\ 0 & ; otherwise \end{cases}$$

Find the density function of $U = \sqrt{X^2 + Y^2}$

SECTION C

5.a) Derive Chebyshev's Inequality and show that it leads to the weak law of large numbers.

b) Define geometric distribution of a random variable. Obtain its mean and variance. Also prove the lack of memory property of it.

6.a) Prove that for binomial distribution, variance is less than mean.

b) A manufacturer, who produces medicine bottles, finds that 0.1%of bottles are defective. The bottles are packed in boxes containing 500 bottles, A drug manufacturer buys 100 boxes from the producer of bottles. Using Poisson distribution, find how many boxes will contain (i) no defective, and (ii) at least two defectives.

SECTION D

7. a) Define the Beta distribution of first and second kind. Also find mean and variance of its first kind.

b) For a normal distribution $N(\mu, \sigma^2)$, show that Mean, Mode and Median coincide.

8.a) What is association of attributes? Write a note on the strength of association and how it is measured.

b) Given $X = 4Y + 5$ and $Y = kX + 4$ are the lines of regression of X on Y and Y on X respectively. Show that $0 < 4k < 1$. If $k = \frac{1}{16}$, find the mean of two variables and coefficient of correlation between them.

Exam Code: 221003 **Paper Code: 3222**

(20)

Programme: Master of Science (Mathematics) Semester-III

Course Title: Operations Research-I -

Course Code: MMSL-3335 (Opt-IV)

Time Allowed: 3 Hours **Max Marks: 80**

1) Paper consists of Eight questions of equal marks (16 marks each).
2) Attempt FIVE questions in all by selecting atleast ONE question from each of FOUR sections. FIFTH question may be attempted from any section.
Note: Students can use non- programmable and non-storage type calculator.

SECTION-A

1. (a) Prove that if an *L.P.P* has a feasible solution, then it also has a basic feasible solution. (8)

   (b) If the feasible region of an linear programming problem (L.P.P) is a convex polyhedron, prove that then there exists an optimal solution to L.P.P. and at least one basic feasible solution must be optimal. (8)

2. Using penality or Big M method , *Maximize* $z = 2x_1 + x_2 + 3x_3$, subject to constraints

$$\begin{aligned} x_1 + x_2 + 2x_3 &\leq 5 \\ 2x_1 + 3x_2 + 4x_3 &= 12 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

(16)

SECTION-B

3. Solve the following mixed integer programming problem:
Maximize $z = -3x_1 + x_2 + 3x_3$ subject to the constraints:

$$\begin{aligned} -x_1 + 2x_2 + x_3 &\leq 4 \\ 4x_2 - 3x_3 &\leq 2 \\ x_1 - 3x_2 + 2x_3 &\leq 3 \\ x_1,\ x_2,\ x_3 &\geq 0; x_1 \text{ and } x_3 \text{ are integers} \end{aligned}$$

(16)

4. (a) Prove that if either the primal or dual problem has an unbounded objective function value, then the other problem has no feasible solution. (8)

   (b) Use branch and bound method to solve the following *L.P.P*
   Maximize $z = x_1 + 2x_2$ subject to the constraints

$$\begin{aligned} 6x_1 + 5x_2 &\leq 25 \\ x_1 + 3x_2 &\leq 10 \\ x_1, x_2 &\geq 0 \text{ and are Integers} \end{aligned}$$

(8)

SECTION-C

5. Four professors are each capable of teaching any one of four different courses. Class preparation time in hours for different topics varies from professor to professor and is given in the table below. each professor is assigned only one course. Determine an assignment schedule so as to minimize the total course preparation time for all courses: (16)

Table 1: Table for Question 6

Professor	Linear Programmes	Queueing Theory	Dynamic Programme	Regression Analysis
A	2	10	9	7
B	15	4	14	8
C	13	14	16	11
D	4	15	13	9

6. The following data is given:

$$\begin{array}{cc} & \text{Destinations} \\ & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \text{Sources} \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 2 & 2 & 3 \\ 4 & 1 & 2 \\ 1 & 3 & X \end{bmatrix} \begin{array}{c} \text{Capacities} \\ 10 \\ 15 \\ 40 \end{array} \\ \text{Demands} & \begin{array}{ccc} 20 & 15 & 30 \end{array} \end{array}$$

The cost of shipment from third source to the third destination is not known. How many units should be transported from sources to destinations so that the total cost of transporting all the units to their destinations is a minimum? (16)

SECTION-D

7. (a) Obtain the optimal strategies for both-persons and the value of the game for zero-sum two-person game whose payoff matrix is as follows:

$$\begin{bmatrix} -4 & 3 \\ -7 & 1 \\ -2 & -4 \\ -5 & -2 \\ -1 & -6 \end{bmatrix}$$

(8)

(b) A vessel is to be loaded with stocks of 3 items. Each unit of item i has a weight w_i and r_i. The maximum cargo weight the vessel can take is 5 and the details of the three items are as follows (8)

Table 2: Table for Question 6(a)

i	w_i	r_i
1	1	30
2	3	80
3	2	65

Table 3: Table for Question 8

		Player A	
		A_1	A_2
	B_1	a_{11}	a_{12}
Player B	B_2	a_{21}	a_{22}

8. Prove that for any 2×2 two-person zero-sum game without any saddle point having the payoff matrix for player A given by Table 3
the optimum mixed strategies

$$S_A = \begin{pmatrix} A_1 & A_2 \\ p_1 & p_2. \end{pmatrix} \text{ and } S_B = \begin{pmatrix} B_1 & B_2 \\ q_1 & q_2 \end{pmatrix}$$

are determined by

$$\frac{p_1}{p_2} = \frac{a_{22} - a_{21}}{a_{11} - a_{12}}, \frac{q_1}{q_2} = \frac{a_{22} - a_{12}}{a_{11} - a_{21}}$$

where $p_1 + p_2 = 1$ and $q_1 + q_2 = 1$. The value v of the game is given by

$$v = \frac{a_{11}a_{22} - a_{21}a_{12}}{a_{11} + a_{22} - (a_{12} + a_{21})}$$

(16)