Exam Code: 508402 **Paper Code: 2208**
(20)

Programme: Master of Science (Mathematics) (FYIP)
Semester-II

Course Title: Calculus-II

Course Code: FMAL-2333

Time Allowed: 3 Hours **Max Marks: 80**

Note: Attempt five questions in all, selecting at least one question from each section. The fifth question may be attempted from any section. Each question carries 16 marks.

Section- A

1. (a) Prove that the sequence $\left\{\frac{2n-9}{3n+1}\right\}$ is
   (i) Monotonically increasing
   (ii) Bounded
   (iii) Has the limit 2/3

   (b) If $\{a_n\}$ and $\{b_n\}$ be two sequences such that $\{a_n\}$ converges to a and $\{b_n\}$ converges to b then prove that $\{a_n b_n\}$ converges to (ab) (8,8)

2. (a) State and prove Sandwich theorem.

   (b) Show that the sequence $\{a_n\}$ where $a_n = 1+\frac{1}{3}+\frac{1}{5}+\cdots\ldots\ldots\ldots\ldots\ldots+\frac{1}{2n-1}$ does not converge by showing that it is not a Cauchy sequence. (8,8)

Section- B

3(a) Discuss the convergence or divergence of the series

$$\frac{1}{2\sqrt{1}}+\frac{x^2}{3\sqrt{2}}+\frac{x^4}{4\sqrt{3}}+\frac{x^6}{5\sqrt{4}}+\cdots\ldots\ldots\ldots\ldots$$

(b) State and prove Cauchy Integral Test. (8,8)

4. (a) Discuss the convergence or divergence of the series $\sum\frac{(n!)^2}{(2n)!}x^n$, x> 0

   (b) Discuss the convergence or divergence of the series $\sum\frac{1}{\sqrt{n}}tan\frac{1}{n}$ (8,8)

Section- C

5. (a)State and Prove Leibnitz Test for alternating series.

   (b) Show that the alternating series $\sum(-1)^{n-1}\frac{n}{5^n}$ is convergent. (8,8)

6. (a) Test the absolute convergence of the series $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}cos^2 n\alpha}{n\sqrt{n}}$ $(\alpha\ real)$

(b) Find the radius of convergence and interval of convergence of the power series $\sum_{k=0}^{\infty} \frac{k(x+3)^k}{4^{k+1}}$

(8,8)

Section- D

7.(a) Obtain Fourier series for f(x) = x in $(-\pi, \pi)$

(b) Find a series of sines and cosines of multiples of x, when function is defined as

$$f(x) = \begin{cases} 0 & , -\pi < x \leq 0 \\ \frac{\pi x}{4}, & 0 < x < \pi \end{cases}$$

(8,8)

8. Find a fourier series for the function defined by $\begin{cases} \frac{\pi}{3}, & 0 < x < \frac{\pi}{3} \\ 0, & \frac{\pi}{3} < x < \frac{2\pi}{3} \\ -\frac{\pi}{3}, & \frac{2\pi}{3} < x < \pi \end{cases}$

(16)

Exam Code: 508302 **Paper Code: 2209**

(20)

Programme: Master of Science (Mathematics) (FYIP)
Semester-II

Course Title: Matrices and Theory of Equations

Course Code: FMAL-2334

Time Allowed: 3 Hours **Max Marks: 80**

There are total Eight questions of equal marks (16 marks each), two in each of the four Sections (A-D). Candidates are required to attempt five questions, selecting at least one question from each section. The fifth question may be attempted from any section.

Section A

1. (a) Prove that rank of the transpose of a matrix A is the same as that of the original matrix.

   (b) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & -1 & 3 \\ -2 & -4 & 4 & -7 \\ 1 & 2 & 1 & 2 \end{bmatrix}$ by reducing to normal form.

2. (a) Find the inverse of $\begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ by using elementary operations.

(b) Discuss for all values of k , the system of equations

$$(3k-8)x + 3y + 3z = 0$$
$$3x + (3k-8)y + 3z = 0$$
$$3x + 3y + (3k-8)z = 0$$

Section B

3. (a) Define Characteristic values of a matrix and find characteristic vectors for the matrix given below

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

(b) Using Cayley-Hamilton theorem, find A^8 , 1f A - $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

4. (a) Find the g.c.d of $f(x) = x^4 + 3x^3 + 2x^2 + x + 1$ and $g(x) = x^3 + x^2 + x + 1$ and write it as $a(x)f(x) + b(x)g(x)$.

(b) Solve the equation $f(x) = x^4 + 4x^3 - 6x^2 - 36x - 27 = 0$ Given that it has a repeated root.

Section C

5. (a) If the roots of $x^3 + ax^2 + bx + c$ are in G.P. then $ca^3 = b^3$

b) If α, β, γ are roots of $x^3 - px^2 + qx - r = 0$, find the value of $\sum \left(\frac{\alpha}{\beta} + \frac{\beta}{\gamma}\right)$

6. (a) Discuss the nature of roots of the equation

$x^3 - 6x^2 + 9x - 2 = 0$ and locate them.

(b) Solve the equation $x^4 - x^3 - 30x^2 - 76x - 56 = 0$ by Newton's method of divisors.

(c) Use Newton's Method of approximation to calculate up to 3 places of decimal the cube root of 41.

Section D

7. (a) Use Cardon's method to solve $x^3 - 6x^2 - 6x - 7 = 0$

(b) Use Descarte's method to solve $x^4 + 12x - 5 = 0$

8. (a) Use Ferrari's method to solve

$$2x^4 + 6x^3 - 3x^2 + 2 = 0$$

(b) Find the nature of the roots of the equation

$$x^3 - 6x^2 - 6x - 14 = 0$$

Exam Code: 508402 **Paper Code: 2210**

(20)

Programme: Master of Science (Mathematics) (FYIP)
Semester-II

Course Title: Solid Geometry

Course Code: FMAL-2335

Time Allowed: 3 Hours **Max Marks: 80**

NOTE: Attempt five questions in all selecting at least one from each section. The fifth question may be attempted from any section. All questions carry 16 marks.

Section A

1. (a) Find the condition that three given planes have a common line of intersection 8

(b) Shift the origin to a suitable point so that the equation $x^2 + y^2 + z^2 - 4x - 8y + 6z - 4 + 0$ is transformed into an equation in which the first degree terms are absent. 8

2. (a) Eliminate the mixed terms from the equation

$$5(x - y + z)^2 - 3(x + y)^2 - 4(x - y - 2z)^2 + 3 = 0 \ .$$

8

(b) Find the centre and the radius of the circle

$$x^2 + y^2 + z^2 - 2y - 4z - 11 = 0, \ x + 2y + 2z - 15 = 0.$$

8

Section B

3. (a) Define orthogonal spheres and find the angles of intersection of the spheres $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 2x + 2y$. 8

(b) Find the equation of the tangent plane to the sphere $x^2 + y^2 + z^2 + 6x - 2z + 1 = 0$ which pass through the line $3(16 - x) = 3z = 2y + 30$. 8

4. (a) Find the equation of the radical plane of the spheres

$$x^2 + y^2 + z^2 + 2u_1x + 2v_1y + 2w_1z + d_1 = 0$$
$$x^2 + y^2 + z^2 + 2u_2x + 2v_2y + 2w_2z + d_2 = 0$$

Also prove that the radical plane of two spheres is perpendicular to the line joining their centres. 8

(b) Find the equation of a sphere which belongs to the coaxial system whose limiting points are $(1,2,0), (2,2,0)$ and which passes through the point $(3,-1,0)$. 8

Section C

5. (a) Prove that the equation of the cone, whose vertex is origin , is homogeneous in (x, y, z) and conversely. 8

(b) If a right circular cone has three mutually perpendicular generators, the semi-vertical angle is $tan^{-1}\sqrt{2}$.

6. (a) Find the enveloping cylinder of the sphere $x^2 + y^2 + z^2 + 2x + 2y + 2z + 2 = 0$ whose generators are parallel to the line $\frac{x}{1} = \frac{y}{-1} = \frac{z}{1}$. 8

(b) Find the equation of the right circular cylinder whose guiding circle is $x^2 + y^2 + z^2 - 2x + 4y - 6z - 2 = 0, 2x + 3y + 6z = 0$

Section D

7.(a) Find the equation of the surface of the revolution obtained by rotating the curve $y^2 = 4ax$, $z = 0$ about tangent at the vertex. 8

(b) Find the locus of points from which three mutually perpendicular tangent lines can be drawn to the conicoid $ax^2 + by^2 + cz^2 = 1$ 8

8. (a) Show that the plane $8x - 6y - z = 5$ touches the paraboloid $\frac{x^2}{2} - \frac{y^2}{3} = z$. Also find the coordinates of the point of contact. 8

(b) Find the equation of the enveloping cone from the point (x_1, y_1, z_1) to the central conicoid $ax^2 + by^2 + cz^2 = 1$. 8

Exam Code: 508402 **Paper Code: 2211**
(20)

Programme: Master of Science (Mathematics) (FYIP)
Semester-II

Course Title: Dynamics

Course Code: FMAL-2336

Time Allowed: 3 Hours **Max Marks: 80**

Note: Attempt FIVE questions, selecting at least ONE question from each section. The Fifth question may be attempted from any Section. Each question carries 16 marks.

Section-A

Q1.(a) A string passes over a smooth fixed pulley and to one end is attached a mass m_1 and to the other a smooth light pulley over which passes another string with masses m_2 and m_3 at the ends. If the system is released from rest, show that m_1 will not move if $\frac{4}{m_1}=\frac{1}{m_3}+\frac{1}{m_2}$. 10

(b) A 150 kg man uses a rope to descend from a window, the rope has a breaking strength of 120 kg. What is the maximum time he can use the rope, if the distance to be covered is 20m? 6

Q2(a) A particle moving with a uniform acceleration in a straight line passes about A,B,C. If AB=BC=b and if time from A to B is t_1, B to C is t_2, prove that the acceleration is $\frac{2b(t_1-t_2)}{t_1t_2(t_1+t_2)}$. 8

(b) A 70 kg man carrying a 14 kg suitcase rides in an elevator that has an upward acceleration of 0.14 m/sec^2. Find the pull on his arm due to weight of suitcase and the force with which his feet press on the floor of the elevator. 8

Section-B

Q3(a). A mass m_1 hanging vertically is connected to another mass m_2 placed on a smooth inclined plane of inclination α by means of a light inelastic string passing over a smooth pulley fixed at the top of the plane. The system is released from rest, discuss the motion and find the pressure on the pulley. 10

(b) Two smooth inclined planes of inclinations 30° and 60° respectively are placed back to back and a string, passing over a smooth pulley at the top, joins masses of 0.3 kg and 0.5 kg lying on the planes. Find the acceleration of either mass, the tension in the string and the reactions of the planes. 6

Q4(a)One end of an elastic string whose modulus of elasticity is λ and whose natural length is L, is tied to a fixed point on a smooth horizontal table and the other end is tied to a mass m lying on the table. The particle is pulled to a distance where extension of the string becomes 'a" and then let go; describe the character of the motion. 12

(b) Find the escape velocity of a particle projected from the surface of earth, where g=9.8m/ sec^2 and R=6370km, R being the radius of earth. 4

Section-C

Q5(a)A particle of mass m is projected from a fixed point with velocity u in a direction making an angle $\alpha(\neq \frac{\pi}{2})$ with the horizontal. Neglecting air resistance, find its motion and show that its path is a parabola. 10

(b). A cricket ball thrown by a man from a height of 1.8m at an angle of 30° with the horizontal at a speed of 18m/sec is caught by another field man at a height of 60 cm from the ground. How far were the two men apart? 6

Q6(a) A particle is projected up an inclined plane of inclination β at an elevation α to the horizontal, show that $\tan\alpha = \cot\beta + 2\tan\beta$, if the particle strikes the plane at right angles. 10

(b) A second's pendulum was too long on a given day by a quantity 'a', it was then over corrected so as to become too short by 'a' during the next day. Prove that if L is the correct length, then the number of minutes gained in two days was $1080\frac{a^2}{L^2}$ nearly. 6

Section-D

Q7(a) A particle of mass m falls from rest at a height h above the ground. Show that throughout the motion, the sum of kinetic and potential energies is constant. 10

(b) b) A mass of 4kg falls vertically through a distance of 40m and comes to rest after penetrating 10.19 cm into a soft bed of mud. What is the resistance offered by the bed of mud? 6

Q8(a) An elastic string of natural length L is extended by an amount a, when it supports a mass M at rest, and is extended by an amount b when it is rotating as a conical pendulum, carrying a particle of the same mass, with angular velocity Ω , prove that

$$gb = \Omega^2 a(L + b)$$ 12

(b). Calculate the power required in lifting up a weight of 100kg. with

(i) constant speed of 3m/sec

(ii) uniform acceleration $0.5m/sec^2$ upwards when the velocity is 3m/sec. 4

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KMV-II [N.S.B]
EVE: - 25-05-2024

Exam Code: 508402 **Paper Code: 2212**
(20)

Programme: Master of Science (Mathematics) (FYIP)
Semester-II

Course Title: Modern Physics

Course Code: FMAM-2396

Time Allowed: 3 Hours **Max Marks: 60**

Instruction:

Attempt FIVE questions, selecting at least ONE question from each section. The fifth question may be attempted from any section. Each question carries equal 12 marks.

SECTION - A

1. (a) Define De Brogile's hypothesis and derive an expression for De Broglie's wavelength. 4
   (b) Discuss Davission and Germer experiment in detail. 8
2. (a) Derive an expression for Heisenberg uncertainty principle. 6
   (b) Obtain an expression for Bragg's law of crystal diffraction. 6

SECTION-B

3. (a) Discuss in detail the radioactive dating. 8

(b) Discuss biological effects of radiation. 4

4. (a) What are radio isotopes? Give the applications of radioisotopes in medicines, agriculture and geology. 9

(b) Define half life, mean life and disintegration constant of a radioactive substance. 3

SECTION-C

5. Discuss in detail principle, working and uses of GM counter. 12

6. Write notes on

(a) cloud chamber and

(b) photographic emulsions 12

SECTION-D

7. Discuss in detail the various fundamental interactions in nature. 12

8. Write notes on

(a) Isospin 4

(b) Strangeness 4

(c) Parity 4

Exam Code: 508402 **Paper Code: 2213**

(20)

Programme: Master of Science (Mathematics) (FYIP)

Semester-II

Course Title: Computer Fundamentals and Introduction to 'C' Programming Language

Course Code: FMAM-2137

Time Allowed: 3 Hours **Max Marks: 50**

Instruction:

Attempt five questions, selecting at least one question from each section. The fifth question may be attempted from any section.

SECTION - A

1. What is computer? Explain its block diagram. (10)
2. What is algorithm? Explain with an example. Write its advantages and limitations. (10)

SECTION - B

3. (a) What is identifier? Explain various rules for naming identifier.

   (b) Explain various logical operators.

   (5x2=10)

4. What is data type? Explain with examples along with range and size of each data type. (10)

SECTION - C

5. What is loop? Explain its types along with syntax and example. (10)

6. What is function? How it is declared and defined? What is return type, formal and actual argument in function? (10)

SECTION - D

7. a) Explain multidimensional array? Explain with an example. (10)

8. a) Write a program to transpose a matrix.

   b) What is string? How it is declared and defined in C Programming? (5x2=10)